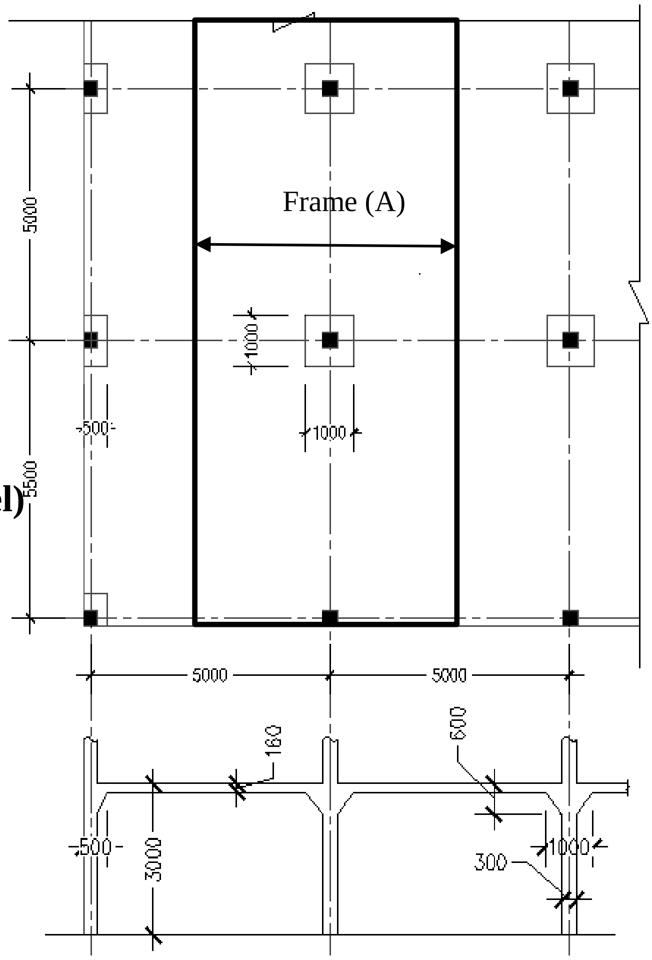
Example 10: For the transverse interior frame (A) of the flat slab floor shown in figure, and by using the Direct Design Method, find:

a. Longitudinal distribution of the total static moment at factored loads.
b. Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

q_u =14.8 kN/m^2
slab thickness = 160 mm
column dimensions = 300 × 300 mm



Solution:

ℓ_2 = 5m

ℓ_n = 5.5– 0.5 – 0.15 = 4.85 m **(exterior panel)**

ℓ_n = 5 – 1 = 4 m **(interior panel)**

q_u = 14.8 kN/m^2

$$M_o = \frac{W_u \ell_2 \ell_n^2}{8}$$

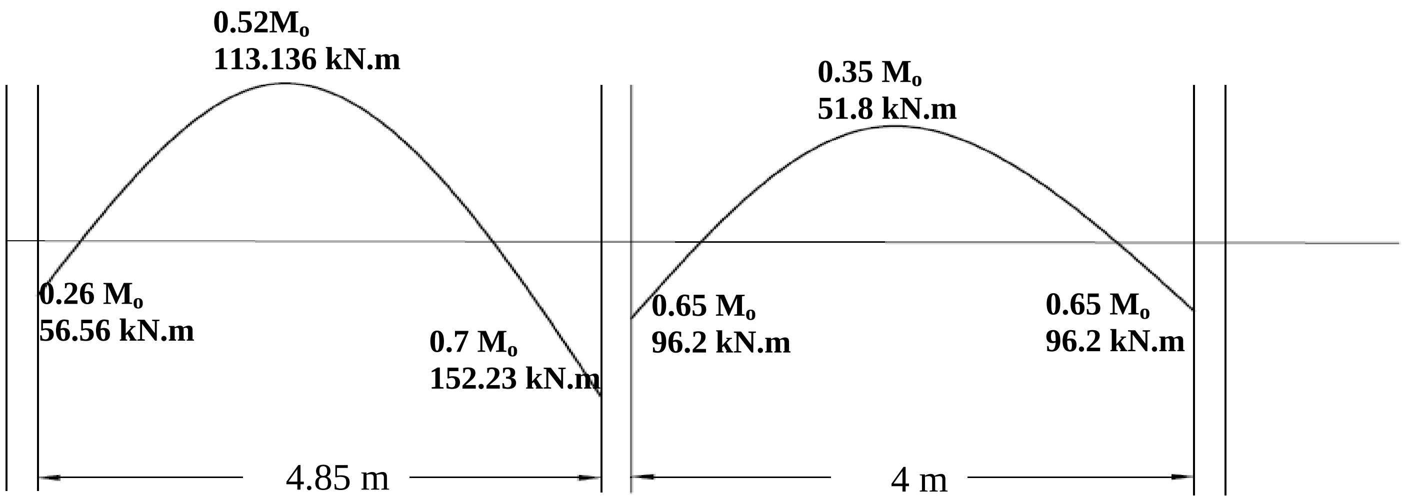
$$M_o = \frac{14.8 \times 4.85^2 \times 5}{8} = 217.57 \text{ kN.m}$$ **(Exterior panel)**

$$M_o = \frac{14.8 \times 4^2 \times 5}{8} = 148 \text{ kN.m}$$ **(interior panel)**

a. Longitudinal distribution of total static moment

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports: Without edge beam	Slab without beams between interior supports: With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



b. Lateral distribution of moment at ***exterior panel*** (column and middle strip moments at exterior support)

Negative moment at interior support

Total negative moment = 152.23 kN.m

Negative moment at column strip = $152.23 \times 0.75 = 114.1725$ kN.m

Negative moment at middle strip = $152.23 - 114.1725 = 38.057$ kN.m

Positive moment

Total moment = 113.136 kN.m

Negative moment at column strip = $113.136 \times 0.60 = 67.8816$ kN.m

Negative moment at middle strip = $113.136 - 67.8816 = 45.2544$ kN.m

Negative moment at exterior support

Total negative moment = 56.56 kN.m

Negative moment at column strip = $1 \times 56.56 = 56.56$ kN.m

Negative moment at middle strip = $56.56 - 56.56 = 0$ kN.m ■

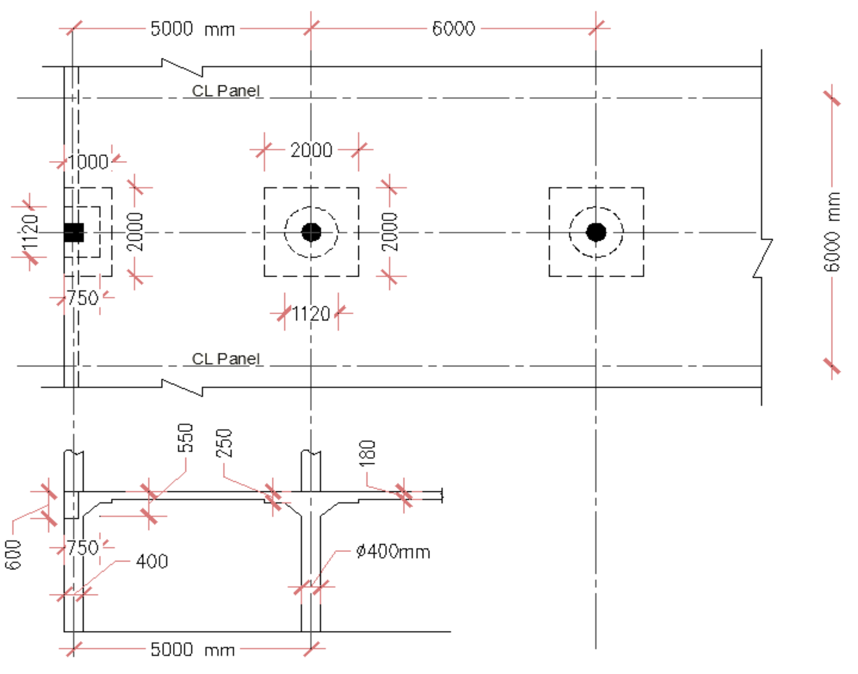
<u>Example 11:</u> For the longitudinal frame of the flat slab floor shown in figure, and by using the Direct Design Method, find:

a. Longitudinal distribution of the total static moment at factored loads.

b. Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

$q_u = 18.0$ kN/m^2

edge beams: 300×600 mm



Solution:

$\ell_2 = 6$ m

$\ell_n = 5 - \frac{0.89 \times 1.12}{2} - 0.55 = 3.95$ m **(exterior panel)**

$\ell_n = 6 - 0.89 \times 1.12 = 5$ m **(interior panel)**

$q_u = 18$ kN/m^2

$$M_o = \frac{W_u \ell_2 \ell_n^2}{8}$$

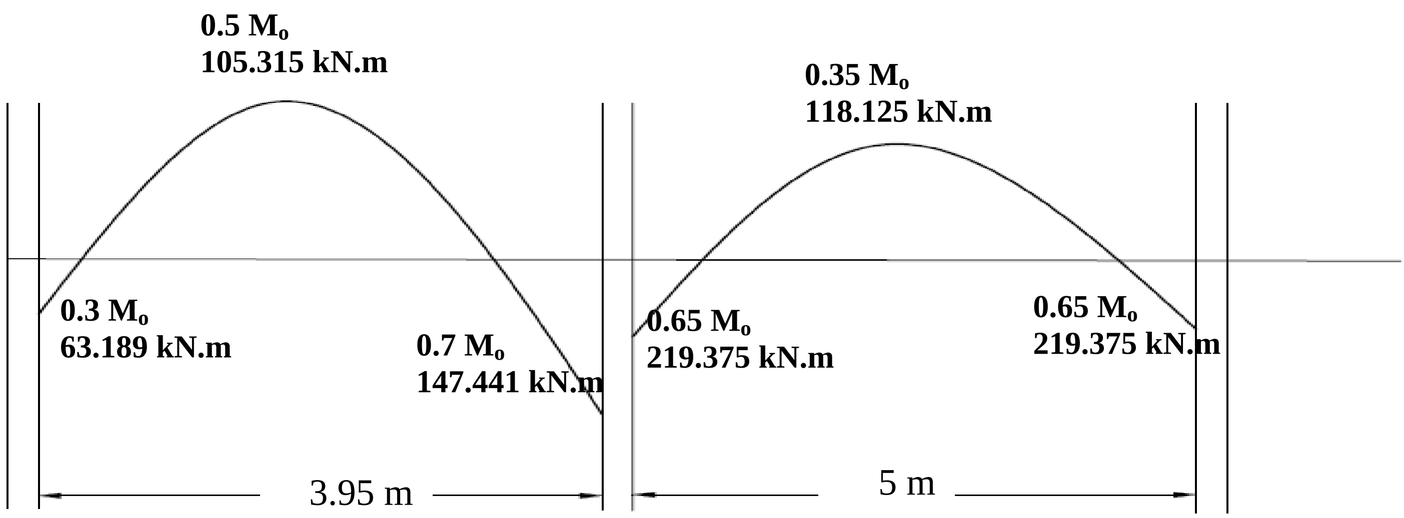
$$M_o = \frac{18 \times 3.95^2 \times 6}{8} = 210.63 \text{ kN.m}$$ **(exterior panel)**

$$M_o = \frac{18 \times 5^2 \times 6}{8} = 337.5 \text{ kN.m}$$ **(interior panel)**

1. Longitudinal distribution of the total static moment at factored loads

Table 8.10.4.2—Distribution coefficients for end spans

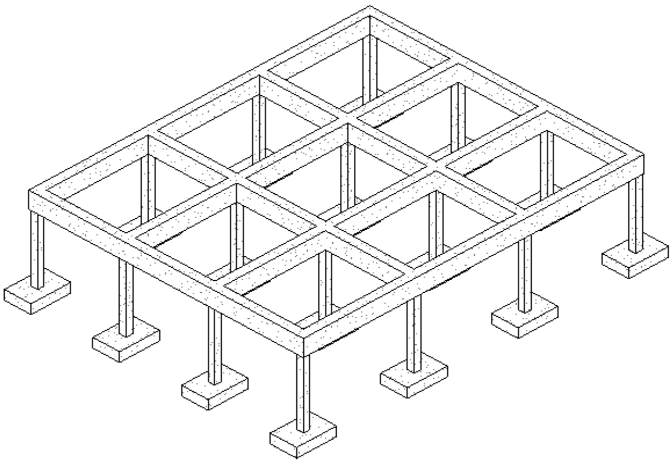
	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports: Without edge beam	Slab without beams between interior supports: With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



b. Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

Homework!

Analysis of Two Way Slab with Interior Beams between Supports



- All the previous examples were slab without interior beams between supports.
- When there are interior beams between supports, Column strip moment will be distributed to **beam moment** and **slab moment** as shown below according to ACI criteria.
- One would ask, can the column strip moment in flat plate or flat slab with edge beam be distributed to beam and slab moments?
  - As long as there are beam (edge beams in this case), the column strip will be distributed to beam and slab moment.

Distribution The Moment In Column Strip To Beam And Slab ACI Code (8.10.5.7.1)

- Beam between supports shall be proportioned to resist ***85% of column strip*** moment if ($\boldsymbol{\alpha_{f1}\frac{\ell 2}{\ell 1}}$) is **equal** to or greater than 1.

Table 8.10.5.7.1—Portion of column strip M_u in beams

$\alpha_{f1}\ell_2/\ell_1$	Distribution coefficient
0	0
≥1.0	0.85

Note: Linear interpolation shall be made between values shown.

- For values ($\boldsymbol{\alpha_{f1}\frac{\ell_2}{\ell_1}}$) between **1** to **zero**, proportion of column strip moment resisted by beam shall be obtained by linear interpolation between 85% and zero percent.

$$\mathbf{M_{beam} = 0.85\left(\alpha_{f1}\frac{\ell 2}{\ell 1}\right) \times M_{C.S}}$$

- If $\alpha_{f1}\frac{\ell_2}{\ell_1}$ >1 use it equal to 1.

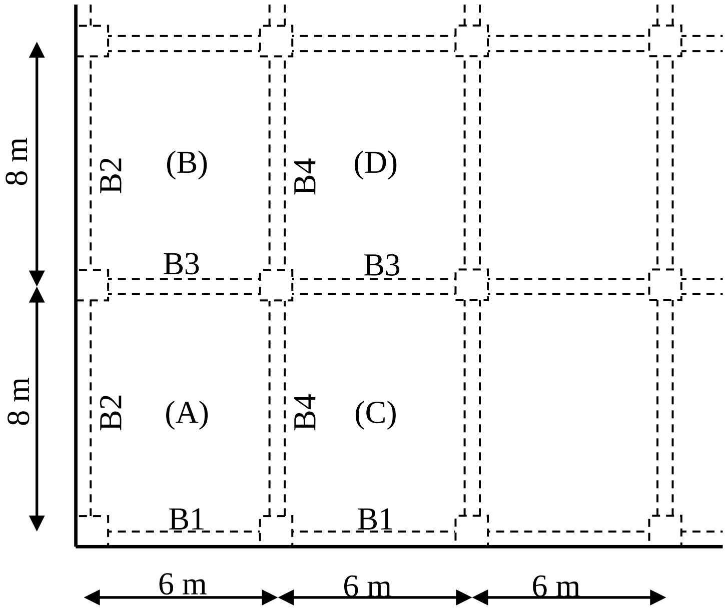
$$\mathbf{M_{c.s} = M_{slab} + M_{beam}}$$

Example

Assume all interior beams are (B3, B4) 300 × 600 mm

- B1 (300 × 600 mm)
- B2 (300 × 700 mm)
- All columns are 600 × 600 mm
- Slab thickness = 180 mm
- Assumed live load = 4.25 kN/m^2
- Dead load =7.58 kN/m^2

  Ib for B1 = 7.992 ×10^9 mm^4
  Ib for B2 = 13.26 ×10^9 mm^4
  Ib for B3 = 9.504 ×10^9 mm^4
  Ib for B4 = 9.504 ×10^9 mm^4



For longitudinal exterior frame find:

1. Longitudinal distribution of static moment at factored loads.
2. Lateral distribution interior and exterior panels

Solution

Computing α_f

Compute the ratio of the flexural stiffness of the longitudinal beams to that of the slab (α_f) in the equivalent rigid frame, for all beams around panels A, B, C, and D.

Beam1

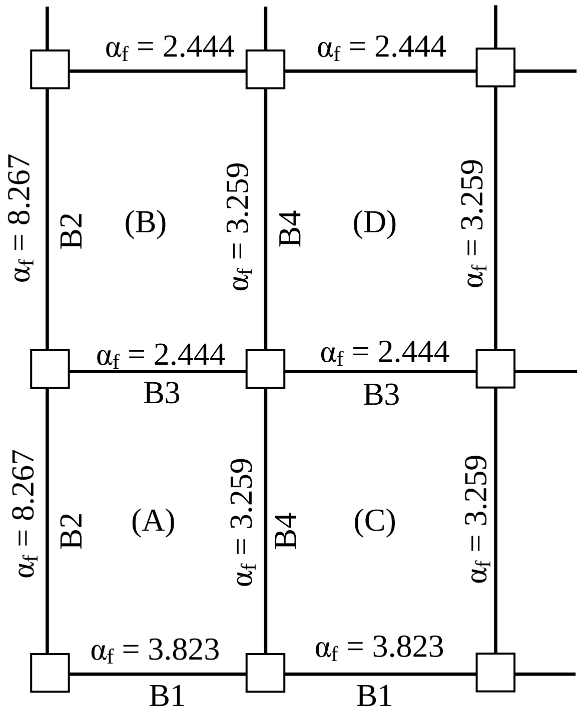
$$\alpha_f = \frac{I_b}{I_s}$$

Ib for B1 = 7.992 ×10^9 mm^4

$$\ell_s = \frac{8000}{2} + \frac{600}{2} = 4300 \text{ mm}$$

$$Is = \frac{\ell_s h^3}{12} = \frac{4300 \times 180^3}{12} = 2.09 \times 10^9 \text{ mm}^4$$

$$\alpha_f = \frac{7.992 \times 10^9}{2.09\ \times 10^9} = 3.823$$



Beam 2

Ib for B2 = 13.26 ×10^9 mm^4

$$\ell_s = \frac{6000}{2} + \frac{600}{2} = 3300 \text{ mm}$$

$$Is = \frac{\ell_s h^3}{12} = \frac{3300 \times 180^3}{12} = 1.604 \times 10^9 \text{ mm}^4$$

$$\alpha_f = \frac{13.26 \times 10^9}{1.604 \times 10^9} = 8.267$$

Beam 3

Ib for B3 = 9.504×10^{9} mm^4

$\ell_s = \frac{8000}{2} + \frac{8000}{2} = 8000$ mm

$$Is = \frac{\ell_s h^3}{12} = \frac{8000 \times 180^3}{12} = 3.888 \times 10^9 \text{ mm}^4$$

$$\alpha_f = \frac{9.504 \times 10^9}{3.888 \times 10^9} = 2.444$$

Beam 4

Ib for B4 = 2.916×10^{9} mm^4

$\ell_s = \frac{6000}{2} + \frac{6000}{2} = 8000$ mm

$$Is = \frac{\ell_s h^3}{12} = \frac{6000 \times 180^3}{12} = 9.504 \times 10^9 \text{ mm}^4$$

$$\alpha_f = \frac{9.504 \times 10^9}{2.916 \times 10^9} = 3.259$$

Exterior longitudinal frame

D.L. = 7.58 kN/m^2

L.L. = 4.25 kN/m^2

$q_u = 1.2 \times 7.58 + 1.6 \times 4.25 = 15.9$ kN/m^2

$$\ell_2 = \frac{8000}{2} + \frac{600}{2} = 4300 \text{ mm}$$

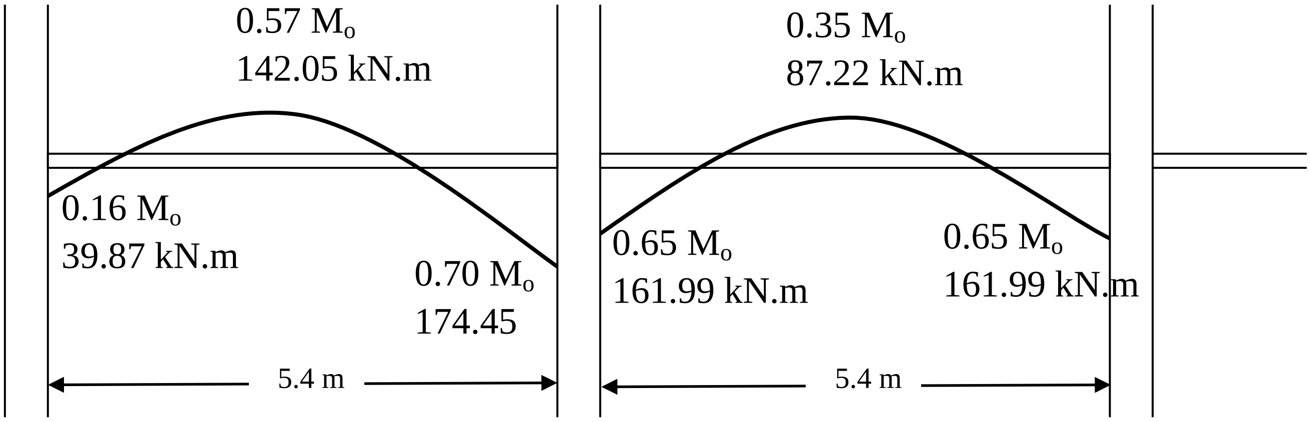
$\ell_n = 6000 - 600 = 5400$ mm $> 0.65\ell_1 = 0.65 \times 6 = 3.9$ m

$$M_o = \frac{1}{8} q_u \ell_2 \ell_n^2 = \frac{1}{8} \times 15.9 \times 4.3 \times (5.4)^2 = 249.21 \text{ kN.m}$$

Longitudinal distribution of moments:

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports: Without edge beam	Slab without beams between interior supports: With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



2. Lateral distribution of longitudinal moments

Interior span

Negative moment (total = -0.65 M_o = -161.99 kN.m)

Negative moment in **column Strip** = 161.99 × 0.65 = 105.29 kN.m

Negative moment in **middle strip** = 161.99 – 105.29 = 56.7 kN.m

Negative moment in **beam** = 105.29 × 0.85 = 89.50 kN.m

Negative moment in column strip **slab** = 105.29 – 89.5 = 15.79 kN.m

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$$\frac{\ell_2}{\ell_1} = \frac{8}{6} = 1.33$$

$$\alpha_f\frac{\ell_2}{\ell_1} = 3.823 \times 1.33 = 5.08$$

Table 8.10.5.1—Portion of interior negative M_u in column strip

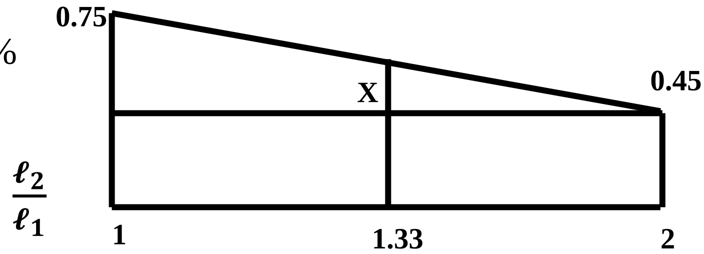
$\frac{\ell_2}{\ell_1} = 1.33$

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1 0.5	1.0	2.0
0	0.75	0.75	0.75
$\boxed{\geq 1.0}$	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

$$\frac{0.75-0.45}{2-1} = \frac{X}{2-1.33}$$

$$0.2 = \frac{X}{0.67} \quad \therefore X = 0.2$$



-Interior C.S $_{\text{coefficient}}$ **%**=0.45 + 0.2 = 0.65%

∴ **-Interior C.S** $_{\text{coefficient}}$ = 0.65

Or by using equation

-Interior C.S $_{\text{coefficient}}$ **%**=75 + 30($\overset{1}{\cancel{\alpha_{f1}\frac{\ell_2}{\ell_1}}}$) × (1− $\frac{\ell_2}{\ell_1}$) = 75 + 30 × $(1-1.33)$ = 65 %

Portion of Column Strip to Beam

Table 8.10.5.7.1—Portion of column strip M_u in beams

$\alpha_{f1}\ell_2/\ell_1$	Distribution coefficient
0	0
≥1.0	0.85

Note: Linear interpolation shall be made between values shown.

∴Column strip portion to beam = 0.85

Positive moment (total = 0.35 M_o = 87.22 kN.m)

Moment in **column strip** = 87.22 × 0.65 = 56.69 kN.m

Moment in **middle strip** = 87.22 – 56.69 = 30.53 kN.m

Moment in **beam** = 56.69 × 0.85 = 48.19 kN.m

Moment in column strip **slab** = 56.69 – 48.19 = 8.5 kN.m

$$\frac{\ell_2}{\ell_1} = \frac{8}{6} = 1.33$$

$$\alpha_f \frac{\ell_2}{\ell_1} = 3.823 \times 1.33 = 5.08$$

$$\frac{0.75-0.45}{2-1} = \frac{X}{2-1.33}$$

$$0.2 = \frac{X}{0.67} \quad \therefore X = 0.2$$

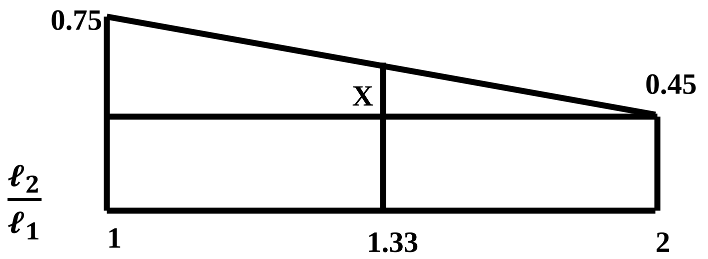
+ C.S $_{\text{coefficient}}$ **%**=0.45 + 0.2 = 0.65%

∴ **+ C.S** $_{\text{coefficient}}$ = 0.65

Table 8.10.5.5—Portion of positive M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1 ($\frac{\ell_2}{\ell_1} = 1.33$)		
	0.5	1.0	2.0
0	0.60	0.60	0.60
$\boxed{\geq 1.0}$	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.



Or by using equation:

+C.S $_{\text{coefficient}}$ **%** = 60 + 30($\alpha_{f1}\frac{\ell_2}{\ell_1}$) × (1.5– $\frac{\ell_2}{\ell_1}$) = 60 + 30 × 1 × (1.5 – 1.33) = 65 %

∴ **+ C.S** $_{\text{coefficient}}$ = 0.65

End span

Negative moment at exterior support (total = -0.16 M_o= -39.87 kN.m)

need $\frac{\alpha_{f1}\ell_2}{\ell_1}$, β_t , and $\frac{\ell_2}{\ell_1}$

Here $\alpha_{f1} = \alpha_{fB1} = 3.823$, $\ell_2 = 8000$ mm, $\ell_1 = 6000$ mm

$$\frac{\ell_2}{\ell_1} = \frac{8000}{6000} = 1.333 \quad \& \quad \frac{\alpha_1 \ell_2}{\ell_1} = \frac{3.823 \times 8000}{6000} = 5.10 > 1.0$$

Computing C

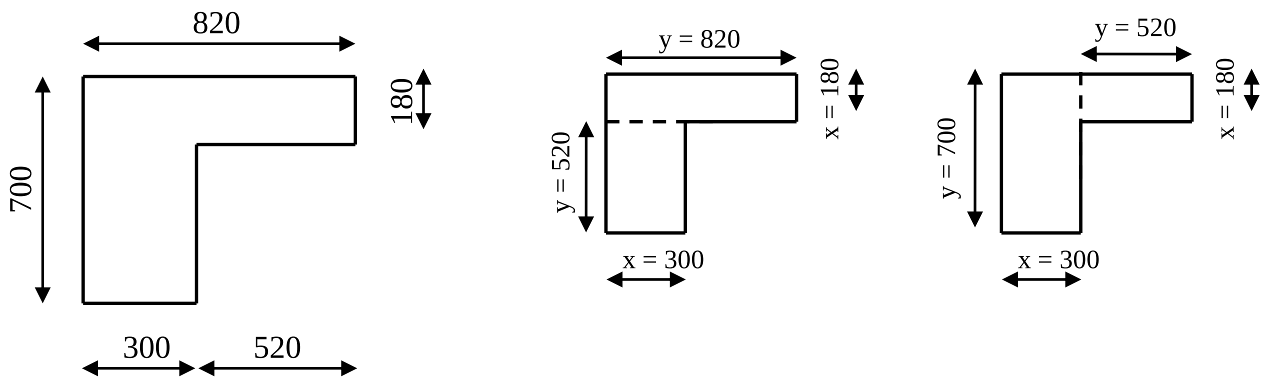
For B2

$$C = \sum \left(1 - 0.63\,\frac{x}{y}\right)\frac{x^3\,y}{3}$$

$$C_1 = \left(1 - 0.63 \times \frac{180}{820}\right)\frac{(180)^3 \times 820}{3} + \left(1 - 0.63 \times \frac{300}{520}\right)\frac{(300)^3 \times 520}{3}$$
$$= 4.353 \times 10^9 \ \text{mm}^4$$

$$C_2 = \left(1 - 0.63 \times \frac{300}{700}\right)\frac{(300)^3 \times 700}{3} + \left(1 - 0.63 \times \frac{180}{520}\right)\frac{(180)^3 \times 520}{3}$$
$$= 5.191 \times 10^9 \ \text{mm}^4$$

∴ For beam B2 $\quad C = 5.191 \times 10^9 \ \text{mm}^4$



$$I_s = \frac{1}{12}\ell_2 h^3 = \frac{1}{12} \times 8000 \times (180)^3 = 3.888 \times 10^9 \ \text{mm}^4$$

$$\beta_t = \frac{C}{2\, I_s} = \frac{5.191 \times 10^9}{2 \times 3.888 \times 10^9} = 0.693$$

$\beta_t = \beta_t = 0.693 \approx 0.69$ (for beam 2)

-Exterior C.S $_{\text{coefficient}}$% $=100 - 10\beta t + 12\ \beta t \left(\cancelto{1}{\alpha_{f1} \frac{\ell_2}{\ell_1}}\right) \times \left(1 - \frac{\ell_2}{\ell_1}\right)$

-Exterior C.S $_{\text{coefficient}}$% $=100 - 10 \times 0.69 + 12 \times 0.69 \times 1 \times (1 - 1.33) = 90.3$ %

-Exterior C.S $_{\text{coefficient}}$% $= 0.903$

∴ Neg. moment in column strip = 39.87 × 0.903 = 36.02 kN.m

Neg. moment in beam = 36.02 × 0.85 = 30.62 kN.m

Neg. moment in column strip slab = 36.02 – 30.62 = 5.4 kN.m

Neg. moment in middle strip = 39.87 - 36.02 = 3.85 kN.m

Positive moments (total = 0.57 M_o = 142.05 kN.m)

Moment in column strip = 142.05 × 0.65 = 92.33 kN.m

Moment in beam = 92.33 × 0.85 = 78.48 kN.m

Moment in column strip slab = 92.33 – 78.48 = 13.85 kN.m

Moment in middle strip = 142.05 – 92.33 = 49.72 kN.m

Interior negative moment (total = 0.70 M_o = -174.45 kN.m)

Moment in column strip = 174.45 × 0.65 = 113.39 kN.m

Moment in beam = 113.39 × 0.85 = 96.38 kN.m

Moment in column strip slab = 113.39 – 96.38 = 17.01 kN.m

Moment in middle strip = 174.45 – 113.39 = 61.06 kN.m

Moments in Exterior longitudinal frame

Total width = 4.3 m, column strip width = 1.8 m, & half middle strip width = 2.5 m.

	Exterior span			Interior span	
	Exterior negative	Positive	Interior negative	Negative	Positive
Total moment (kN.m)	-39.87	+142.05	-174.45	-161.99	+87.22
Moment in beam (kN.m)	-30.62	+78.48	-96.38	-89.50	+48.19
Moment in column strip slab (kN.m)	-5.4	+13.85	-17.01	-15.79	+8.50
Moment in middle strip slab (kN.m)	-3.85	+49.72	-61.06	-56.70	+30.53

H.W for **longitudinal interior frame** finds:

1. Longitudinal distribution of total static moment at factored loads.
2. Lateral distribution for interior and exterior panels